6 - Venn diagrams

Example 1. In a standard deck of 52 cards, how many cards are a red face card or a black number card?

Answer: 3 (Red J, Q, K) + 10 (A, 2, ..., 10) = 13. There are no overlaps.

Example 2. Let $A = \{x \in \mathbb{N} : 12 \le x \le 32\}$, $B = \{x \in \mathbb{N} : -37 \le x \le -7\}$. Find $|A \cup B|$.

Answer: |A|=32-12+1=21, |B|=-7-(-37)+1=31, so |A|+|B|=21+31=52 since there is no overlap.

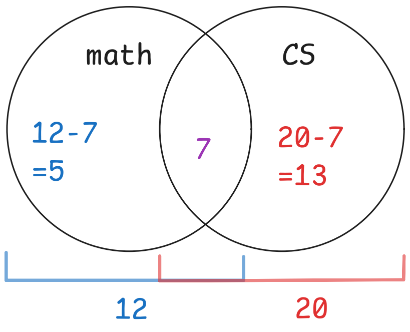
Definition. Sets A and B are disjoint if $A \cap B = \varnothing$, in which case $|A \cup B| = |A| + |B|$

Example 3. How many 2-digit numbers are not divisible by 5?

Answer: U = {2-digit numbers} = {10, 11, ..., 99} => |U| = 99-10+1 = 90.
A = {2-digit numbers divisible by 5} = {10, 15, ..., 95}
=> |A|=95/5-10/5+1=18
Notice A and its complement U-A are disjoint so |U| = |A| + |U-A|
=> # of 2-digit numbers not divisible by 5 = |U-A| = |U| - |A| = 90-18 = 72.

What if sets overlap?

Example 4. In this class, 12 students major in at least math, 20 major in at least CS, and 7 students major in at least math and CS.
(a) How many major in math but not CS? 12-7=5
(b) How many major in CS but not math? 20-7=13
(c) How many major in math or CS?
(12-7)+7+(20-7)=12+20-7=25

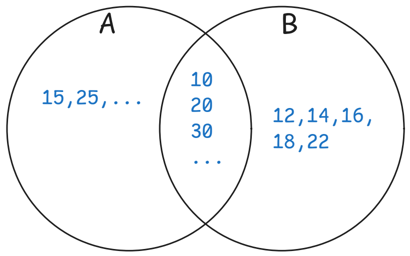


Principle of inclusion-exclusion. (PIE) $|A \cup B| = |A| + |B| - |A \cap B|$

Example 5. How many 2-digit numbers are divisible by 5 or by 2?

Answer:
A = {2-digit numbers divisible by 5} => |A|=18
B = {2-digit numbers divisible by 2} => |B|=4x9=45
A∩B = {... divisible by 10} => |A∩B| = 9

So |A U B| = 18+45-9 = 63-9 = 54



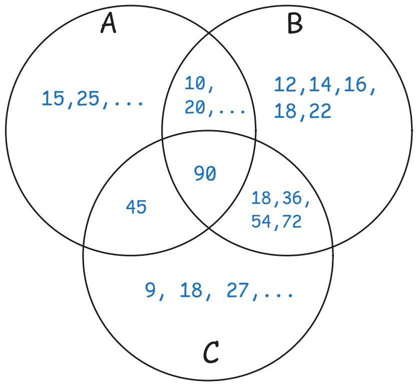
Answer 2: (Use complements)
U = {2-digit numbers} => |U|=90
C = {2-digit numbers are not divisible by 5 nor 2} = {11, 13, 17, 19, etc.}
=> |C| = 4 x 9 = 36.

=> |U-C| = 90 - 36 = 54

Principle of inclusion-exclusion. (PIE)

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Example 6. How many 2-digit numbers are divisible by 2, 5, or 9?



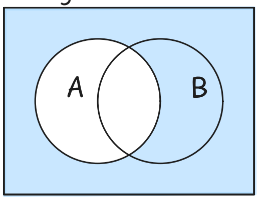
Answer: A, B as before.
C = {2-digit numbers divisible by 9} = {18,27,...,90,99}
=> |C|=10.

|A∩C| = |{45,90}| = 2
|B∩C| = |{18,36,54,72,90}| = 5
|A∩B∩C| = |{90}| = 1

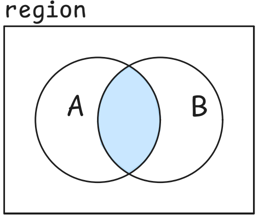
Final Answer: 18+45+10-2-5+1 = 67.

Generic Venn diagrams show relationships that hold for all sets:

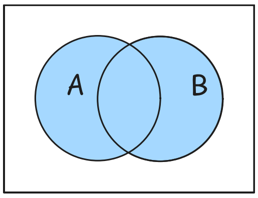
~A: shade everything not in A



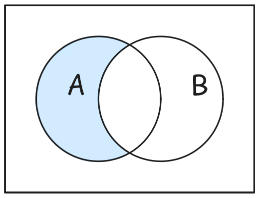
A∩B: shade common region



A∪B: shade A, also shade B

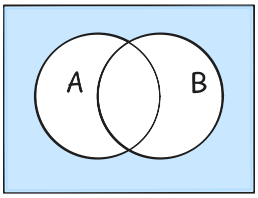


A-B: shade part of A not in B



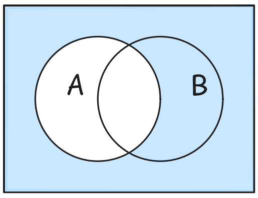
Example 7. Show De Morgan's law ~(A∪B)=(~A)∩(~B) via (generic) Venn diagrams.

~(A∪B)



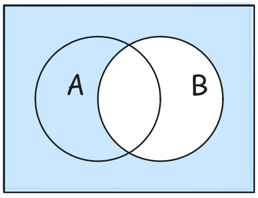
?=

~A



∩

~B



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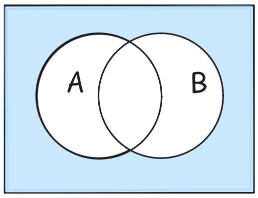
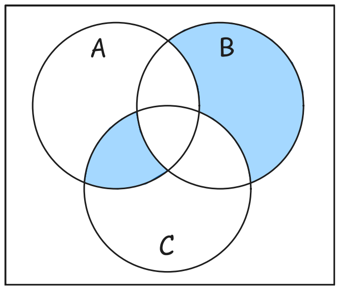
(~A)∩(~B)



Example 8. Put the Venn diagram (at right) into set notation.



Write as {left part} U {right part}
A point in left part is in A and not in B and in C, so
  {left region} = A ∩ ~B ∩ C
A point in right part is not in A and in B and not in C, so
  {right region} = A ∩ ~B ∩ C

Exercises.

1. Show the other De Morgan's law ~(A∩B)=(~A)∪(~B) via (generic) Venn diagrams.
2. How many natural numbers less than 100 do not end with a 0?
3. How many standard playing cards are red or have a number (A, 2, 3,...,10)?
4. How many numbers taken from 1,2,3,...,100, when written out in English, do not contain the letter "x"? (E.g. 1 = "one" has no letter "x".)